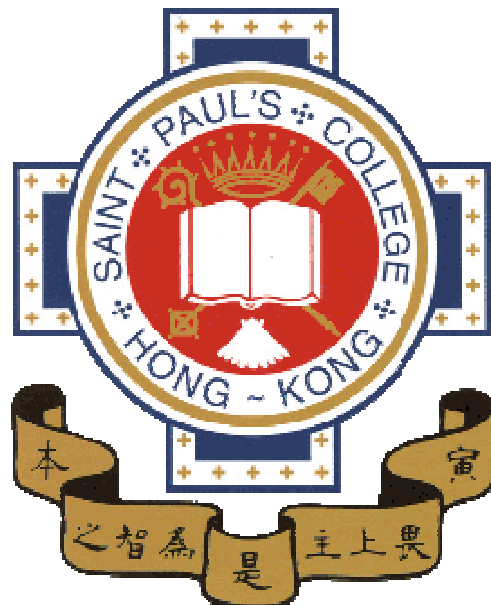


ST. PAUL'S COLLEGE

F.4 Final Examination 2018-2019

Maths II

Multiple Choice Qns



Founded 1851

Suggested Solutions

Time allowed: 50 mins

Paper II

Answers:

1A **2B** **3D** **4C** **5D**
6A **7B** **8A** **9A** **10C**

11D **12D** **13B** **14C** **15A**
16B **17D** **18C** **19D** **20B**

21C **22B** **23D** **24A** **25C**
26B **27C** **28A** **29B** **30C**

A-7, **B-8,** **C-8,** **D-7**

Question 1:

Simplify $\frac{(a \cdot \sqrt{b})^{-2}}{(a^{-1} \cdot \sqrt[3]{b^2})^3}$.

A. $\frac{a}{b^3}$

B. $\frac{a^2}{b^3}$

C. $\frac{a}{b^{10}}$

D. $\frac{1}{a^5 b^{10}}$

Solutions:

Ans.: **A**

$$\begin{aligned} & \frac{(a \cdot \sqrt{b})^{-2}}{(a^{-1} \cdot \sqrt[3]{b^2})^3} \\ &= \frac{a^{-2} \cdot b^{-1}}{a^{-3} \cdot b^2} \\ &= \frac{a^{-2-(-3)}}{b^{2-(-1)}} \\ &= \frac{a}{b^3} \end{aligned}$$

Question 2:

If $x = 5 - \frac{3}{y-4}$, then $y =$

A. $\frac{4x-17}{x-5}$

B. $\frac{4x-23}{x-5}$

C. $\frac{4x+17}{x+5}$

D. $\frac{4x+23}{x+5}$

Solutions:

Ans.: **B**

$$\begin{aligned} x &= 5 - \frac{3}{y-4} \\ \frac{3}{y-4} &= 5-x \\ \frac{y-4}{3} &= \frac{1}{5-x} \\ y-4 &= \frac{3}{5-x} \\ y &= 4 + \frac{3}{5-x} \\ y &= \frac{4(5-x)+3}{5-x} \\ y &= \frac{23-4x}{5-x} \\ y &= \frac{4x-23}{x-5} \end{aligned}$$

Question 3:

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{y}{x} - \frac{x}{y}}(y-x) =$$

- A. 0 B. 1 C. $\frac{1}{x+y}$ D. $-\frac{x-y}{x+y}$

Solutions:

Ans.: **D**

$$\begin{aligned} \cdot \frac{\frac{1}{x} - \frac{1}{y}}{\frac{y}{x} - \frac{x}{y}} \cdot (y-x) &= \frac{\frac{y-x}{xy}}{\frac{y^2-x^2}{xy}} \cdot (y-x) \\ &= \frac{(y-x)}{(y-x)(y+x)} \cdot (y-x) = -\frac{x-y}{x+y} \end{aligned}$$

Question 4:

The L.C.M. of a^2+3a+2 , a^2+4a+4 and a^3+8 is

- A. $a+2$ B. $(a+1)(a+2)^3$
C. $(a+1)(a+2)^2(a^2-2a+4)$ D. $(a+1)(a+2)^2(a^2+2a+4)$

Solutions:

Ans.: **C**

$$\begin{aligned} \cdot a^2+3a+2 &= (a+1)(a+2) \\ \cdot a^2+4a+4 &= (a+2)^2 \\ \cdot a^3+8 &= (a+2)(a^2-2a+4) \\ \cdot \text{L.C.M.} &= (a+1)(a+2)^2(a^2-2a+4) \\ \cdot \text{H.C.F.} &= (a+2) \end{aligned}$$

Question 5:

If $\frac{A}{x+1} + \frac{B}{x-1} \equiv \frac{1-5x}{x^2-1}$, then $A =$

- A. 3 B. 2 C. -2 D. -3

Solutions:

Ans.: **D**

$$\begin{aligned} \cdot \frac{A(x-1)+B(x+1)}{(x+1)(x-1)} &= \frac{1-5x}{x^2-1} \\ \cdot A+B &= -5 \quad \dots \text{(1)} \\ \cdot -A+B &= 1 \quad \dots \text{(2)} \\ \text{(1)-(2): } 2A &= -6 \\ A &= -3 \end{aligned}$$

Alternatively,

$$A(x-1)+B(x+1) \equiv 1-5x$$

Put $x = -1$,

$$A(-1-1)+B(0) \equiv 1-5(-1)$$

$$A = -3$$

Question 6:

Let $f(x) = 2x^3 + ax^2 + bx - 3$. If $f\left(\frac{1}{2}\right) = 0$ and $f(-1) = 0$, then $f(x) =$

A. $(x+1)(2x-1)(x+3)$

B. $(x+1)(2x-1)(x-3)$

C. $(x-1)(2x+1)(x-3)$

D. $(x-1)(2x+1)(x+3)$

Solutions:

Ans.: **A**

. Method 1

▪ $f\left(\frac{1}{2}\right) = 0$ means $(2x-1)$ is a factor of $f(x)$.

▪ $f(-1) = 0$ means $(x+1)$ is a factor of $f(x)$.

so $f(x) = 2x^3 + 7x^2 + 2x - 3$
 $= (x+1)(Ax+B)(x+3)$

By comparing the coefficient of x^3 : $A = 2$

By comparing the constant term : $B = -1$

so $f(x) = 2x^3 + 7x^2 + 2x - 3$
 $= (x+1)(2x-1)(x+3)$

. Method 2

▪ $f\left(\frac{1}{2}\right) = 0$

$$2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) - 3 = 0$$

▪ $a + 2b = 11 \dots (1)$

▪ $f(-1) = 0$

$$2(-1)^3 + a(-1)^2 + b(-1) - 3 = 0$$

▪ $a - b = 5 \dots (2)$

(1)-(2) : $3b = 6$

$$b = 2$$

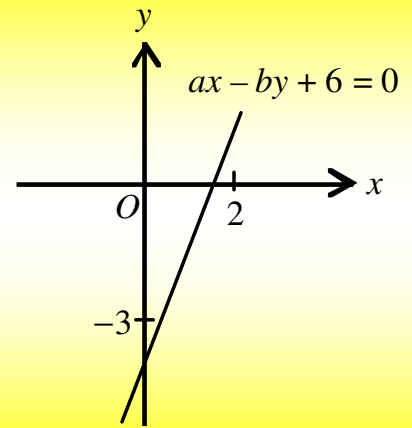
$$a = 7$$

▪ $f(x) = 2x^3 + 7x^2 + 2x - 3 = (x+1)(2x^2 + 5x - 3) = (x+1)(2x-1)(x+3)$

Question 7:

The figure shows the graph of the straight line $ax - by + 6 = 0$. Which of the following are true?

- (I) $a < b$
 (II) $a > -3$
 (III) $b > -2$
- A. I and II only
 B. I and III only
 C. II and III only
 D. I, II and III



Solutions:

Ans.: **B**

• The line $L: ax - by + 6 = 0$

• Slope of line $L = \frac{a}{b} > \frac{2}{3}$ [I is **true**]
 $a < \frac{2}{3}b < b$

Alternatively,

Taking x-intercept = 1 and y-intercept = -4,

Equation of the line is

Multiplying by 1.5, equation becomes

Rearranging

Comparing with

gives

$$y = 4x - 4$$

$$1.5y = 6x - 6$$

$$-6x + 1.5y + 6 = 0$$

$$ax - by + 6 = 0$$

$$a = -6, b = -1.5$$

$$a < b, a < -3 \text{ and } b > -2$$

[I, III are correct, II is not.

• x-intercept of $L = -\frac{6}{a} < 2$

$$-6 > 2a \quad [\text{since } a < 0]$$

$$-3 > a \quad [\text{II is NOT true}]$$

• y-intercept of $L = \frac{6}{b} < -3$

$$6 > -3b \quad [\text{since } b < 0]$$

$$-2 < b \quad [\text{III is true}]$$

Question 8:

If the straight lines $cx + dy = 6$ and $2x - 4y - 3 = 0$ are perpendicular to each other and intersect at a point on the y-axis, then $c =$

- A. -16 B. -4 C. 4 D. 16

Solutions:

Ans.: **A**

• For $L_1 \perp L_2$, $(-\frac{c}{d})(-\frac{2}{-4}) = -1$,

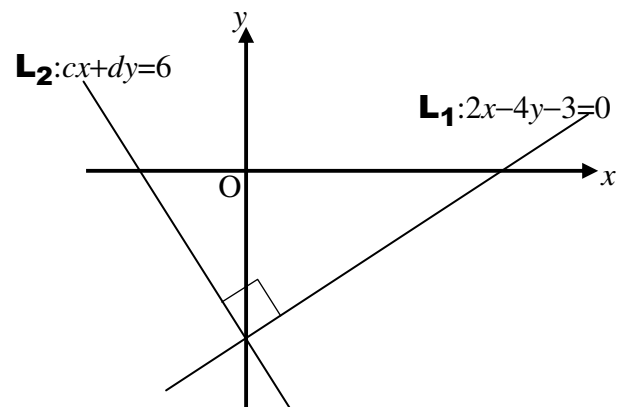
$$c = 2d$$

• For L_1 : $x=0$, $y = -\frac{3}{4}$,

• For L_2 : $c(0) + d(-\frac{3}{4}) = 6$,

$$d = -8$$

$$c = 2d = -16$$



Question 9:

If $\alpha \neq \beta$ and $\begin{cases} 2\alpha^2 + b\alpha - c = 0 \\ 2\beta^2 + b\beta - c = 0 \end{cases}$, then $\alpha^2 + \alpha\beta + \beta^2 =$

A. $\frac{b^2 + 2c}{4}$

B. $\frac{b^2 - 2c}{4}$

C. $\frac{c^2 - 2b}{4}$

D. $b^2 - c$

Solutions:

Ans.: **A**

- α, β are roots of the quadratic equation $2x^2 + bx - c = 0$.

$$\alpha + \beta = -\frac{b}{2}$$

$$\alpha \times \beta = -\frac{c}{2}$$

- $$\begin{aligned} \alpha^2 + \beta\alpha + \beta^2 &= (\alpha + \beta)^2 - \alpha\beta \\ &= \left(-\frac{b}{2}\right)^2 - \left(-\frac{c}{2}\right) \\ &= \frac{b^2}{4} + \frac{c}{2} \\ &= \frac{b^2 + 2c}{4} \end{aligned}$$

Question 10:

Let $f(x) = 4x - kx^2 - 3$. If the graph of $y + k = f(x)$ does not cut the x -axis.

Which of the following values is a possible value/are possible values of k ?

(I) -5

(II) 1

(III) 3

A. I only

B. III only

C. I, III only

D. II, III only

Solutions:

Ans.: **C**

- $$\begin{aligned} y &= f(x) - k \\ &= -kx^2 + 4x - (k+3) \quad \text{has NO } x\text{-intercept} \end{aligned}$$

- $$\begin{aligned} \Delta &= (4)^2 - 4(-k)[-(k+3)] < 0 \\ &= -k^2 - 3k + 4 < 0 \end{aligned}$$

- Check with calculator :**

- $$-(-5)^2 - 3(-5) + 4 < 0 \quad \text{[I is true]}$$

- $$-(1)^2 - 3(1) + 4 = 0 \quad \text{[II is NOT true]}$$

- $$-(3)^2 - 3(3) + 4 < 0 \quad \text{[III is true]}$$

Question 11:

Which of the following statements about the graph of $y = (2 - x)(x - 4) - 8$ is true ?

- A. The graph cuts the x -axis at two distinct points.
- B. The graph opens upwards.
- C. The y -intercept of the graph is -8 .
- D. The coordinates of the vertex of the graph are $(3, -7)$.

Solutions:

Ans.: **D**

A. On the x -axis, $y=0$,

$$0 = (2-x)(x-4) - 8$$

$$0 = -x^2 + 6x - 16$$

$$0 = x^2 - 6x + 16,$$

$$\Delta = (-6)^2 - 4(1)(16)$$

$$< 0, \text{ no solution}$$

NO x -intercept for the curve.

B. Coefficient of $x^2 = -1 < 0$,
the curve **opens downward**

C. On the y -axis, $x=0$,

$$y = (2 - 0)(0 - 4) - 8$$

$$= -16$$

D. $y = (2-x)(x-4) - 8$

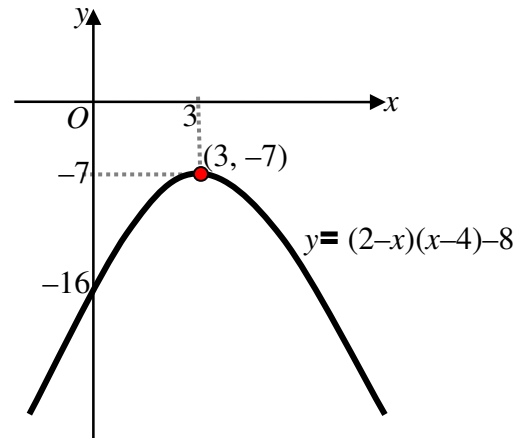
$$= -x^2 + 6x - 16$$

$$= -[x^2 - 6x + 3^2 - 3^2] - 16$$

$$y = -[(x-3)^2 - 9] - 16$$

$$= -(x-3)^2 - 7,$$

Vertex =



Alternatively,

For $y = ax^2 + bx + c$;

Vertex = (h, k) where

$$h = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$$

$$k = -3^2 + 6(3) - 16 = -7$$

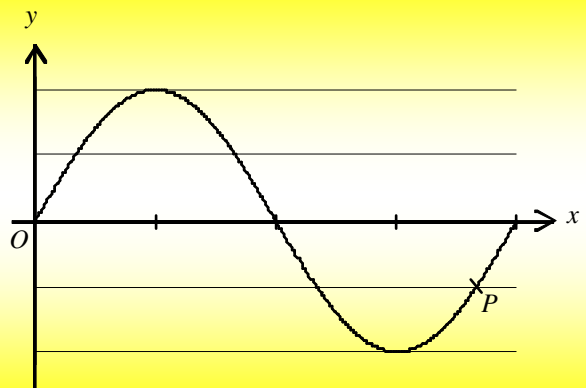
so Vertex = $(3, -7)$

Question 12:

The figure shows the graph of $y = \sin(x^\circ)$.

Which of the following is most probably the x -coordinate of point P ?

- A. 135
- B. 150
- C. 315
- D. 330



Solutions:

Ans.: **D**

$$y = \sin(330^\circ) = -0.5$$

Question 13:

For $0^\circ \leq x < 360^\circ$, how many roots does the equation $3\sin^2 x = \cos x + 1$ have ?

- A. 2 B. 3 C. 4 D. 5

Solutions:

Ans.: **B**

$$\begin{aligned} \cdot \quad 3(1 - \cos^2 x) &= \cos x + 1 \\ 3\cos^2 x + \cos x - 2 &= 0 \\ (3\cos x - 2)(\cos x + 1) &= 0 \\ \cos x &= \frac{2}{3} \quad \text{or} \quad \cos x = -1 \\ x &= 48.2^\circ, 312^\circ \quad \text{or} \quad 180^\circ \end{aligned}$$

Question 14:

Let $f(x) = 3(kx^2 + 1)$ and $g(x) = 27x^3 + 18x - 5$. If both expressions leave the same remainder when divided by $3x - 2$, then $k =$

- A. -21 B. $-\frac{9}{4}$ C. 9 D. $\frac{21}{2}$

Solutions:

Ans.: **C**

$$\begin{aligned} \cdot \quad f\left(\frac{2}{3}\right) &= g\left(\frac{2}{3}\right) \\ 3\left[k\left(\frac{2}{3}\right)^2 + 1\right] &= 27\left(\frac{2}{3}\right)^3 + 18\left(\frac{2}{3}\right) - 5 \\ k\left(\frac{4}{3}\right) + 3 &= 8 + 12 - 5 \\ k &= 9 \end{aligned}$$

Question 15:

Let $f(x) = (x - 1)(x - 3)g(x) + (x - 2)$, where $g(x)$ is a cubic polynomial. Which of the following are true ?

- (I) The remainder of $f(x) \div g(x)$ is $x - 2$.
(II) The remainder of $f(x) \div (x - 1)(x - 3)$ is $x - 2$.
(III) The remainder of $f(x) \div (x - 1)$ is $x - 2$.

- A. I, II only B. I, III only C. II, III only D. I, II and III

Solutions:

Ans.: **A**

$$\cdot \quad \text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

- (I) The **divisor** is $g(x)$, the remainder can be a linear polynomial.
(II) The **divisor** is $(x - 1)(x - 3)$, the remainder is a linear polynomial.
(III) The **Remainder** is $f(1) = 0 + (1 - 2) = -1$ is a constant.

Question 16:

Given that $\frac{a}{5} = \frac{b}{3} = \frac{c}{2}$ and $x : y : z = 2^a : 3^b : 6^c$. Arrange x, y and z in ascending order.

- A. $x < y < z$ B. $y < x < z$ C. $y < z < x$ D. $z < y < x$

Solutions:

Ans.: B

- Let $a=5k, b=3k, c=2k$,
- $x : y : z = 2^{5k} : 3^{3k} : 6^{2k} = 32^k : 27^k : 36^k$

Question 17:

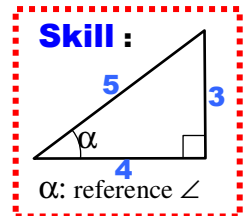
Given that $\tan \theta = \frac{-3}{4}$ and θ lies in the second quadrant. Which of the following quadratic equations has roots $\sin \theta$ and $-2 \cos \theta$?

- A. $x^2 - 11x + 24 = 0$ B. $x^2 + 11x + 24 = 0$
 C. $25x^2 + 55x + 24 = 0$ D. $25x^2 - 55x + 24 = 0$

Solutions:

Ans.: D

- $\tan \theta = -\frac{3}{4}$, θ lies in the **second quadrant**; $\sin \theta = \frac{3}{5}$, $\cos \theta = -\frac{4}{5}$
- Sum** of roots $= \frac{3}{5} + \frac{8}{5} = \frac{11}{5}$; **Product** of roots $= \frac{3}{5} \times \frac{8}{5} = \frac{24}{25}$,
- The required equation**: $x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0$
or $x^2 - (\frac{11}{5})x + (\frac{24}{25}) = 0$ **or** $25x^2 - 55x + 24 = 0$



Question 18:

$$\frac{1}{\cos(360^\circ - \theta)} + \frac{\sin(180^\circ - \theta)}{\tan(270^\circ + \theta)} =$$

- A. 0 B. $-\cos \theta$ C. $\cos \theta$ D. $\frac{\sin^2 \theta}{\cos \theta}$

Solutions:

Ans.: C

$$\begin{aligned} \frac{1}{\cos(360^\circ - \theta)} + \frac{\sin(180^\circ - \theta)}{\tan(270^\circ + \theta)} &= \frac{1}{\cos(\theta)} + \frac{\sin(\theta)}{-\frac{1}{\tan(\theta)}} \\ &= \frac{1}{\cos(\theta)} + \sin(\theta) \div \left[-\frac{1}{\tan(\theta)}\right] \\ &= \frac{1}{\cos(\theta)} + \sin(\theta) \times \left[-\frac{\sin(\theta)}{\cos(\theta)}\right] \\ &= \frac{1 - \sin^2(\theta)}{\cos(\theta)} = \frac{\cos^2(\theta)}{\cos(\theta)} = \cos(\theta) \end{aligned}$$

Question 19:

If a is a real number, then the real part of $\frac{3-i^3}{a-i} + i^6$ is

A. $\frac{3a+1}{a^2-1}$

B. $\frac{3a-1}{a^2+1}$

C. $\frac{-a^2+3a+2}{a^2-1}$

D. $\frac{-a^2+3a-2}{a^2+1}$

Solutions:

Ans.: **D**

$$\begin{aligned} \cdot \quad \frac{3-i^3}{a-i} + i^6 &= \frac{3-(i^2)(i)}{a-i} + (i^2)^3 &= \frac{3+(i)}{a-i} - 1 \\ &= \left(\frac{3+i}{a-i}\right)\left(\frac{a+i}{a+i}\right) - 1 \\ &= \frac{3a+ai+3i+i^2}{a^2-i^2} - 1 \\ &= \frac{3a+ai+3i-1-a^2}{a^2+1} - 1 \\ &= \frac{(-a^2+3a-2)+(a+3)i}{a^2+1} \\ &= \left(\frac{-a^2+3a-2}{a^2+1}\right) + \left(\frac{a+3}{a^2+1}\right)i \end{aligned}$$

$$\cdot \quad \text{The real part of } \frac{3-i^3}{a-i} + i^6 = \frac{-a^2+3a-2}{a^2+1}$$

Question 20:

A sum of \$100 000 is deposited at an interest rate of 6% per annum for n years compounded monthly. The interest earned correct to the nearest dollar is \$12 716.

Find n correct to the nearest $\frac{1}{12}$.

A. $1\frac{8}{12}$

B. 2

C. $2\frac{1}{12}$

D. $2\frac{2}{12}$

Solutions:

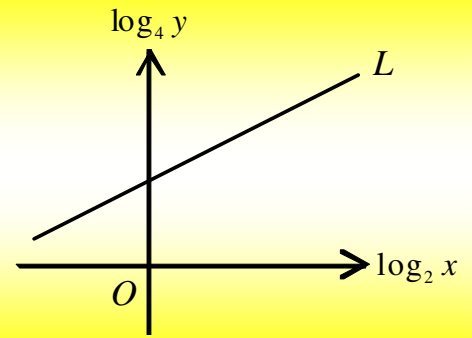
Ans.: **B**

$$\begin{aligned} \cdot \quad 100000 \left[\left(1 + \frac{0.06}{12}\right)^{12n} - 1 \right] &= 12716 \\ (1.005)^{12n} &= 1.12716 \\ \log[(1.005)^{12n}] &= \log(1.12716) \\ 12n &= \frac{\log(1.12716)}{\log(1.005)} \\ 12n &= 24.00004 \\ n &= 2 \end{aligned}$$

Question 21:

In the figure, the straight line L shows the relation between $\log_2 x$ and $\log_4 y$. It is given that L passes through the points $(4, 4)$ and $(8, 7)$. If $y = kx^a$, then $k =$

- A. $\frac{3}{2}$ B. 1
C. 4 D. 8



Solutions:

Ans.: **C**

- Slope of the straight line L = $\frac{7-4}{8-4} = \frac{3}{4}$
- L passes through $(0, c)$:
 $\frac{c-4}{0-4} = \frac{3}{4}$, $c = 1$, the y-intercept

Method 1

Equation of L :

$$\log_4 y = \frac{3}{4}(\log_2 x) + 1 \quad [\text{In the form } Y = mX + c]$$

$$\frac{\log(y)}{\log(4)} = \left(\frac{3}{4}\right) \frac{\log(x)}{\log(2)} + 1$$

$$\frac{\log(y)}{2\log(2)} = \left(\frac{3}{4}\right) \frac{\log(x)}{\log(2)} + 1$$

$$2[\log(y)] = 3[\log(x)] + 4[\log(2)]$$

$$\log(y^2) = \log(x^3) + \log(2^4)$$

$$\log(y^2) = \log[(2^4)(x^3)]$$

$$y^2 = (2^4)(x^3)$$

- $y = (2^2)(x^{1.5})$

$$k = 4, \quad a = 1.5$$

Method 2

$$y = k(x^a)$$

$$\log_4(y) = a[\log_4(x)] + \log_4(k)$$

$$\log_4(y) = (a) \frac{\log(x)}{\log(4)} + \log_4(k)$$

$$\log_4(y) = (a) \frac{\log(x)}{2\log(2)} + \log_4(k)$$

$$\log_4(y) = \left(\frac{a}{2}\right) \log_2(x) + \log_4(k)$$

- $\log_4(k) = 1 =$ the y-intercept

$$k = 4^1, \quad a = 1.5$$

Question 22:

Let a and b be positive constants. Denote the graph of $y = \log_a x + b$ by G .
The x -intercept of G is 4 and G passes through the point (16, 2), then

- A. $x = 2^{8-2y}$ B. $x = 2^{y+2}$ C. $x = 2^y + 2$ D. $x = 2^y + 4$

Solutions:

Ans.: **B**

- $y = \log_a(x) + b$
- $0 = \log_a(4) + b$ (1) [G passes through (4, 0)]
- $2 = \log_a(16) + b$ (2) [G passes through (16, 2)]

• **(2)-(1):**

$$2 = \log_a\left(\frac{16}{4}\right)$$

$$a^2 = 4$$

$$a = 2,$$

$$b = -2$$

- $y = \log_2(x) - 2$
- $y+2 = \log_2(x)$
- $2^{y+2} = x$

Question 23:

Let O be the origin. The coordinates of the points P and Q are (30, 10) and (-10, -30) respectively. The x -coordinate of the circumcentre of $\triangle OPQ =$

- A. -10 B. 0 C. 10 D. 25

Solutions:

Ans.: **D**

- $M = \left(\frac{30-10}{2}, \frac{10-30}{2}\right) = (10, -10)$

• Equation of L_1 :

$$y = -x$$

- $N = \left(\frac{30+0}{2}, \frac{10+0}{2}\right) = (15, 5)$

- Slope of $OP = \frac{10-0}{30-0} = \frac{1}{3}$

• Equation of L_2 :

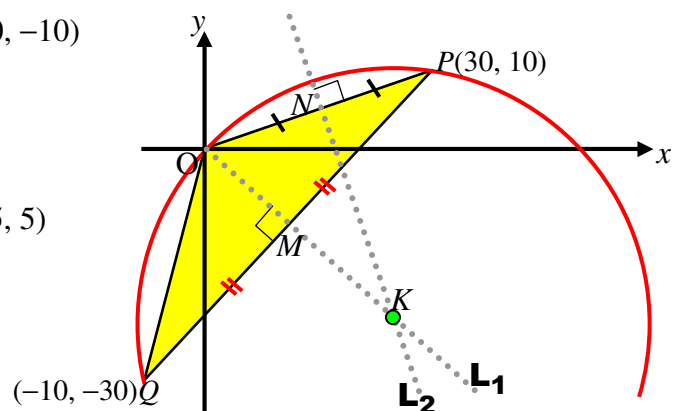
$$y-5 = (-3)(x-15)$$

$$y = -3x + 50$$

• Solving equations of L_1 and L_2 :

$$x = 25$$

$$y = -25, \quad K = (25, -25)$$



Alternatively,
Sketch the graph,
see which of the options fits.

Question 24:

Find the maximum value of $\cos^2 x - 4\cos x + 7$ for $0^\circ \leq x \leq 360^\circ$

A. 12

B. 8

C. 7

D. 4

Solutions:

Ans.: **A**

$$\cdot (\cos x)^2 - 4(\cos x) + 7$$

$$= (\cos x)^2 - 4(\cos x) + 4 + 3$$

$$= (\cos x - 2)^2 + 3$$

$$\cdot \text{Maximum of } (\cos x)^2 - 4(\cos x) + 7 = (-1 - 2)^2 + 3 = 12$$

Question 25:

In the figure, $\triangle ABC$ cuts the circle at D, E, F and G . M and N are mid-points of DE and FG respectively and $ME = NF$.

O is the centre of the circle,
 $\angle ABC = 66^\circ$ and $\angle ACB = 72^\circ$.

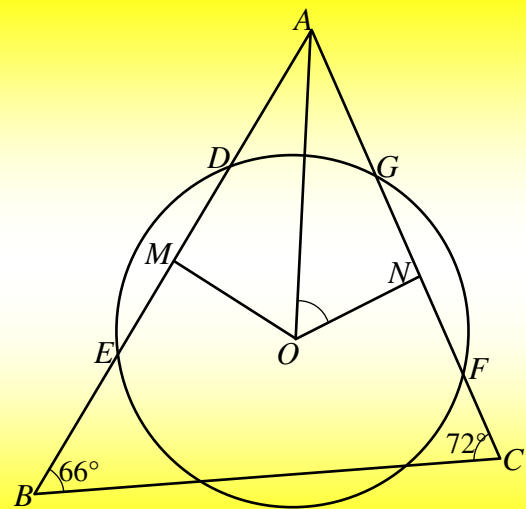
$\angle AON =$

A. 84°

B. 72°

C. 69°

D. 66°



Solutions:

Ans.: **C**

$$\cdot \angle ONA = 90^\circ \quad (\text{Line joining centre to the mid-pt. of chord } \perp \text{ chord})$$

$$\angle OMA = 90^\circ \quad (\text{Line joining centre to the mid-pt. of chord } \perp \text{ chord})$$

$$\cdot ME = NF \quad (\text{Given})$$

$$2 \times ME = 2 \times NF$$

$$DE = GF \quad (M, N \text{ are given to be mid-points})$$

$$\cdot OM = ON \quad (\text{Equal chords, equidistant from centre})$$

$$\cdot OA = OA \quad (\text{Common})$$

$$\triangle OMA \cong \triangle ONA \quad (\text{R.H.S.})$$

$$\cdot \angle OAM = \angle OAN \quad (\text{corr. } \angle\text{s, } \cong \Delta\text{s})$$

$$\angle OAM + \angle OAN + 72^\circ + 66^\circ = 180^\circ \quad (\angle \text{ sum of } \Delta)$$

$$\angle OAN = 21^\circ$$

$$\cdot \angle AON + 90^\circ + 21^\circ = 180^\circ \quad (\angle \text{ sum of } \Delta)$$

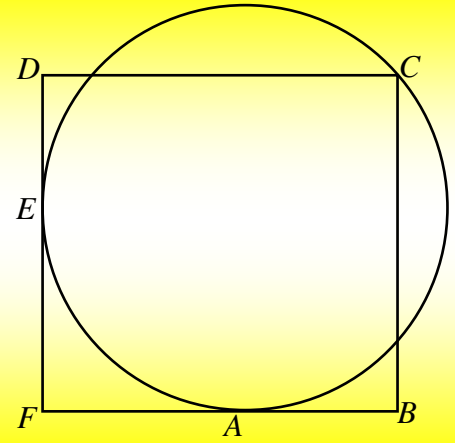
$$\angle AON = 69^\circ$$

Question 26:

In the figure, rectangle $BCDF$ touches the circle at A and E , C is a point on the circumference, $FB = 9$, $FD = 8$,

the radius of the circle =

- A. $\frac{72}{17}$
- B. 5
- C. 6
- D. cannot be determined



Solutions:

Ans.: B

- Let the radius be r .

$$(8-r)^2 + (9-r)^2 = r^2$$

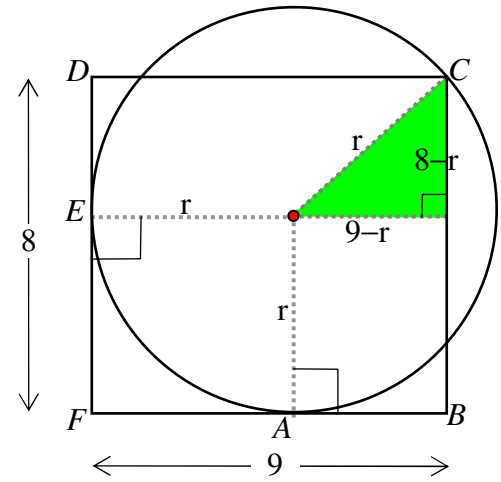
$$64 - 16r + r^2 + 81 - 18r + r^2 = r^2$$

$$r^2 - 34r + 145 = 0$$

$$(r-5)(r-29) = 0$$

$$r = 5 \quad \text{or} \quad r = 29$$

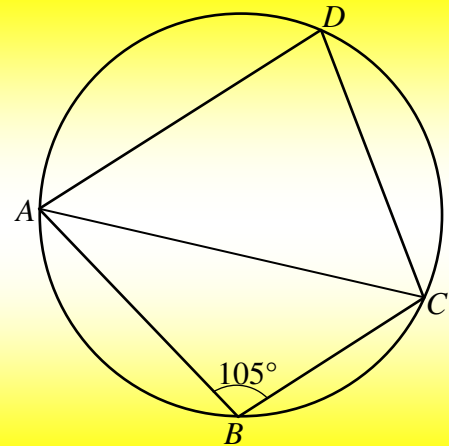
(rejected)



Question 27:

In the figure, BD is a diameter, $\widehat{AB} : \widehat{BC} = 3 : 2$ and $\angle ABC = 105^\circ$. $\angle DAC =$

- A. 42°
- B. 54°
- C. 60°
- D. 63°



Solutions:

Ans.: C

- Let $\angle ACB = 3k$, $\angle BAC = 2k$

$$3k + 2k + 105^\circ = 180^\circ$$

(\angle sum of Δ)

$$k = 15^\circ$$

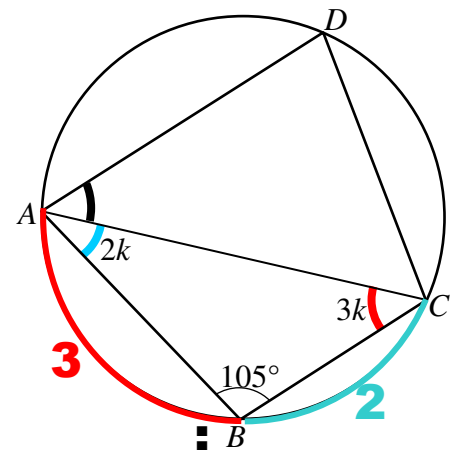
- $\angle DAB = 90^\circ$

(\angle in semi-circle)

$$\angle DAC + \angle BAC = 90^\circ$$

- $\angle DAC = 90^\circ - 2(15^\circ)$

$$= 60^\circ$$

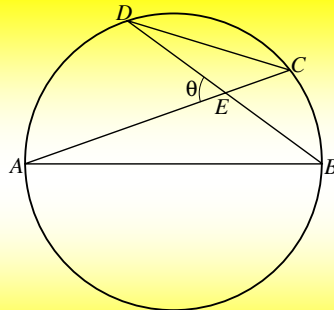


Question 28:

In the figure, AB is a diameter of the circle $ABCD$. It is given that AC and BD intersect at E and $\angle AED = \theta$. Which of the following are true?

- (I) $\triangle DCE \sim \triangle ABE$.
- (II) $\frac{CD}{AB} = \cos \theta$.
- (III) $\angle ABD = \frac{\theta}{2}$.

- A. I, II only B. I, III only C. II, III only D. I, II and III

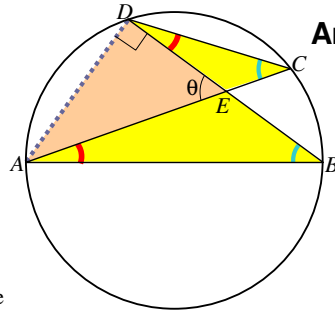


Solutions:

- (I) is **true** (A.A.A.)
- (II) is **true** since

$$\cos \theta = \frac{DE}{AE} \quad [\text{In } \triangle ADE]$$

$$= \frac{CD}{AB} \quad [\text{corr. sides, } \sim \triangle s]$$
- (III) is **NOT true** since E is not given to be the centre of the circle

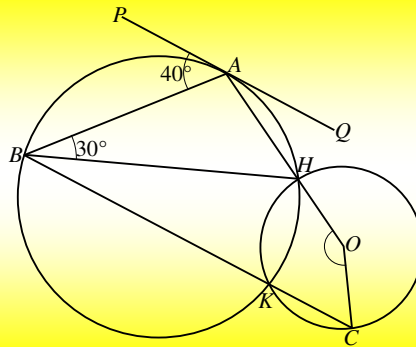


Ans.: **A**

Question 29:

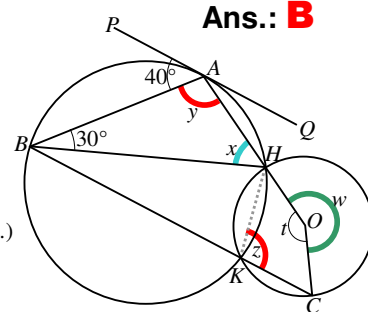
In the figure, O is the centre of the smaller circle, the two circles intersect at H and K , OHA and CKB are straight lines. PQ touches the larger circle at A . $\angle PAB = 40^\circ$ and $\angle ABH = 30^\circ$. $\angle COH =$

- A. 20°
 B. 140°
 C. 110°
 D. 70°



Solutions:

- $x = 40^\circ$ ($\angle s$ in alternate segment)
- $y = 180^\circ - 40^\circ - 30^\circ = 110^\circ$ (\angle sum of Δ)
- $z = y = 110^\circ$ (ext. \angle , cyclic quad.)
- $w = 2 \times z = 220^\circ$ (\angle at centre = $2 \times \angle$ at the circum.)
- $t = 360^\circ - 220^\circ = 140^\circ$ ($\angle s$ at a pt.)



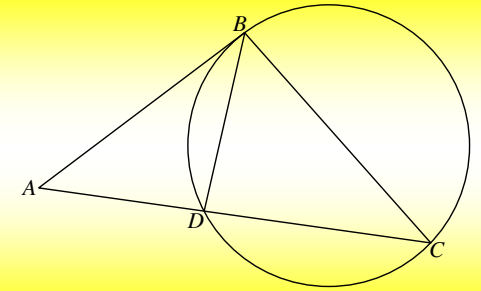
Ans.: **B**

Question 30:

In the figure, AB is the tangent to the circle at B and ADC is a straight line.

If $AC : AB = 3 : 2$, then the area of $\triangle ABD$: the area of $\triangle BCD =$

- A. 2 : 3
 B. 3 : 5
 C. 4 : 5
 D. 4 : 9



Solutions:

- $x = y$ ($\angle s$ in alternate segment)
- $\angle BAC = \angle DAB$ (common)
- $\angle C = \angle D$ (common)
- $\triangle ABC \sim \triangle ADB$ (A.A.A.)
- $AC : AB = 3 : 2$ (Given)

$$\frac{AC}{AB} = \frac{3}{2}$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ADB} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\frac{\text{Area of } \triangle BCD}{\text{Area of } \triangle ADB} = \frac{9-4}{4} = \frac{5}{4}$$

$$\frac{\text{Area of } \triangle ADB}{\text{Area of } \triangle BCD} = \frac{4}{5}$$

$$\text{Area of } \triangle ABD : \text{Area of } \triangle BCD = 4 : 5$$

Ans.: **C**

