ST. PAUL'S COLLEGE

F.4 MidYr Examination 2019-2020

Maths II

Multiple Choice Qns

Founded 1851

Suggested Solutions

Time allowed: 60 mins

Paper II Answers:

1C	2A	3C	4B	5D
6C	7B	8A	9A	10C
11D	12C	13D	14D	15B
16A	17D	18D	19B	20A
21B	22B	23C	24A	25C
26B	27C	28D	29D	30

A:6, B:7, C:8, D:8

Question 1:

$0.3\dot{4}5\dot{} =$

A. 0.35 (correct to 3 significant figures)
B. 0.345 (correct to 4 decimal places)
C. 0.3455 (correct to 4 significant figures)
D. 0.3454 (correct to 4 decimal places)

Solutions:

Ans.: C

Question 2:

Which of the following **MUST** be true ?

(1) $1.\dot{3}2\dot{7}$ is a rational number.

(2) $\sqrt{\frac{48}{3}}$ is an irrational number.

(3) $\frac{22}{7}$ is an irrational number.

A. (1) only
B. (1) and (2) only
C. (2) and (3) only
D. (1), (2) and (3)

Solutions:

Ans.: A

(1) $1.\dot{3}2\dot{7} = \frac{442}{333}$ is **rational**.

(2) $\sqrt{\frac{48}{3}} = \sqrt{16} = 4$ is **rational**.

(3) $\frac{22}{7}$ is a fraction which is **rational**.

Question 3:

Solve $x^2 = 5x$.

A. $x = 0$
B. $x = -5$
C. $x = 0$ or $x = 5$
D. $x = -5$ or $x = 5$

Solutions:

Ans.: C

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ or $x - 5 = 0$

$x = 0$ or $x = 5$

Question 4:

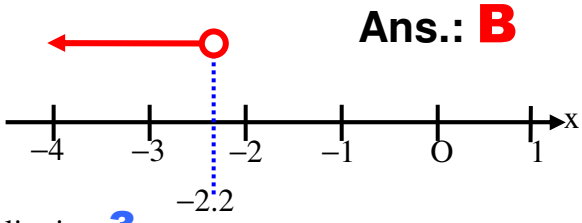
Find the greatest integer that satisfies the inequality $\frac{5x+3}{-2} \geq 4$.

A. -4 B. -3

C. -2 D. 1

Solutions:

Ans.: B



- $5x+3 \leq -8$
  $5x \leq -11$
  $x \leq -2.2$
- **The greatest integer** satisfies the inequality is **−3** .

Question 5:

$\frac{(-2xy^{-1})^{-2}}{3x^{-5}} =$

A. $-\frac{4x^3y^2}{3}$ B. $\frac{4x^3y^2}{3}$

C. $\frac{x^3y^2}{6}$ D. $\frac{x^3y^2}{12}$

Solutions:

Ans.: D

- $\frac{(-2xy^{-1})^{-2}}{3x^{-5}} = \frac{(-2)^{-2}x^{-2}y^2}{3x^{-5}} = \frac{x^5y^2}{(-2)^2 \cdot 3x^2} = \frac{x^3y^2}{12}$
- **OR**

  substitute x=**2**, y=**3**, $\frac{(-2(\mathbf{2})(\mathbf{3})^{-1})^{-2}}{3(\mathbf{2})^{-5}}$ =6 (**by calculator**); A.:−96 B.:96 C.:12 D.:6

Question 6:

$x^2 - 6x - 4y^2 - 12y =$

A. $(x-2y)(x-2y+6)$ B. $(x-2y)(x+2y+6)$

C. $(x+2y)(x-2y-6)$ D. $(x+2y)(x+2y-6)$

Solutions:

Ans.: C

- $x^2 - 6x - 4y^2 - 12y$
  $= x^2 - 4y^2 - 6x - 12y$
  $= [x^2 - (2y)^2] - 6(x + 2y)$
  $= (x-2y)(x+2y) - 6(x + 2y)$
  $= (x+2y)(x-2y - 6)$

OR

- **Substitute** x=**1**, y=**0** into $x^2-6x-4y^2-12y$:
  Value of the question= −5 (**calculator**)
  A.: **7** B.: **7**
- **Substitute** x=**0**, y=**1** into $x^2-6x-4y^2-12y$:
  Value of the question= −16 (**calculator**)
  C.: **−16** D.: **−8**

Question 7:

Make b the subject of the formula $\frac{a+2b}{a}-2=b$.

A. $b=\frac{a+2b}{a}-2$

B. $b=\frac{a}{2-a}$

C. $b=\frac{2-a}{a}$

D. $a=\frac{b+2}{b}$

Solutions:

Ans.: B

. $\frac{a+2b}{a}-2 = b$

$a+2b = a(b+2)$

$2b-ab = a$

$b(2-a) = a$

$b = \frac{a}{2-a}$

OR

. **Substitute** a=**1** into $\frac{a+2b}{a}-2=b$:

$1 + 2b - 2 = b$ giving $b = 1$

. **Substitute** a=**1** & b=1 into A, B, C, D :
Only option B will make **L.H.S.=R.H.S.** !!

Question 8:

If $f(x+2)=3x-5$, then $f(1)$ =

A. -8

B. -5

C. -2

D. 1

Solutions:

Ans.: A

. Replace x by **-1** in " $f(x+2)=3x-5$ " :

$f(\,(\textbf{-1})+2) = 3(\textbf{-1})-5$

$f(1) = -8$

Question 9:

The points $A=(-3, 6)$, $B=(-1, 3)$ and $C=(5, k)$ lie on the same straight line. Find k .

A. -6

B. -2

C. 6

D. 9

Solutions:

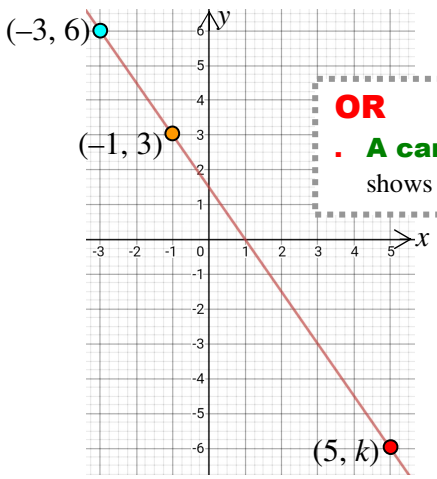
Ans.: A

. Slope of AB = Slope of AC

$\frac{6-3}{-3-(-1)} = \frac{3-k}{-1-5}$

$\frac{3}{-2} = \frac{3-k}{-6}$

$k = -6$



OR

. **A careful construction**
shows that k= –6 !!

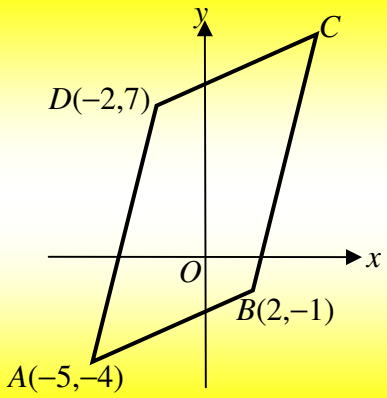
Question 10:

In the figure, $ABCD$ is a parallelogram where $A=(-5,-4)$, $B=(2,-1)$ and $D=(-2,7)$.

Find the slope of AC .

A. $-\frac{7}{5}$ B. $-\frac{5}{7}$

C. $\frac{7}{5}$ D. $\frac{5}{7}$



Solutions:

Ans.: C

. Mid-point of BD = $\left(\frac{2+(-2)}{2}, \frac{-1+7}{2}\right)$

= M = $(0, 3)$

. Slope of AC = Slope of AM

= $\frac{3-(-4)}{0-(-5)}$

= $\frac{7}{5}$

OR

. **A careful construction** of the given points in a proper scaled coordinate system will give the coordinates of M and hence finding the slope of AC.

Question 11:

Given two points $P=(-6, 5)$ and $Q=(3,-4)$ in the x-y plane. If R is a point lying on the line segment PQ such that $4PR=5RQ$, then the coordinates of R are

A. $(-\frac{13}{3}, \frac{40}{9})$ B. $(-\frac{14}{3}, \frac{41}{9})$

C. $(-2, 1)$ D. $(-1, 0)$

Solutions:

Ans.: D

. $PR : RQ$ = $5 : 4$

. R = $\left(\frac{5(3)+4(-6)}{5+4}, \frac{5(-4)+4(5)}{5+4}\right)$ = $(-1, 0)$

OR

. **A careful construction** of the given points in a proper scaled coordinate system will give the coordinates of R .

Question 12:

$i^{2019} =$.

A. -1 B. 1

C. $-i$ D. i

Solutions:

Ans.: C

. i^{2019} = $i^{2018} \cdot i$ = $(i^2)^{1009} \cdot i$

= $(-1)^{1009} \cdot i$ = $-i$

Question 13:

If A, B and C are constants such that $3x^2 + 7 \equiv Ax(x-2) + B(x+2) + C$, then $C =$

A. 5 B. 3

C. −3 D. −5

Solutions:

Ans.: D

- **Method 1 :**
  - $3x^2 + 7 \equiv Ax(x-2) + B(x+2) + C$

    $3x^2 + 0x + 7 \equiv Ax^2 - 2Ax + Bx + 2B + C \equiv Ax^2 + (B-2A)x + (2B+C)$
  - By comparing coefficients :
    - $A = 3,$
    - $B - 2A = 0, \quad B = 6$
    - $2B + C = 7, \quad C = -5$

- **Method 2 :**
  Put $x=0$ and $x=2$ successively for 2 equations in unknowns B and C and solve them.

Question 14:

If β is a root of the equation $x^2+2x-7=0$, then $3\beta^2+6\beta+10 =$

A. −11 B. 43

C. 37 D. 31

Solutions:

Ans.: D

- $\beta^2 + 2\beta - 7 = 0$

  $\beta^2 + 2\beta = 7$
- $3\beta^2 + 6\beta + 10 = 3(7)+10 = \mathbf{31}$

OR **With calculator program :**

- $x \approx \mathbf{1.828}$ or $x \approx \mathbf{3.828}$
- $3\beta^2 + 6\beta + 10 \approx 3(\mathbf{1.828})^2 + 6(\mathbf{1.828}) + 10 \approx 31$

Question 15:

If α and β are the roots of the quadratic equation $x^2+5x+k=0$, then $\alpha^2-5\beta =$

A. $-25-k$ B. $25-k$

C. $-5+k$ D. $5+k$

Solutions:

Ans.: B

- α is a root of $x^2+5x+k=0$ gives

  $\alpha^2+5\alpha+k = 0$

  so $\alpha^2 = -5\alpha-k$
- $\alpha^2-5\beta = (-5\alpha-k)-5\beta$

  $= -5(\alpha+\beta)-k = -5(-5)-k = 25-k$

OR

- **Let** $k=\mathbf{4}$ giving $\alpha=-1, \beta=-4$:

  $\alpha^2-5\beta = (-1)^2-5(-4) = 21$
- **Put** $k=\mathbf{4}$ into options A, B, C, D :

  A.: −29 B.: 21 C.: −1 D.: 9

Question 16:

A quadratic equation with $(3-2i)$ and $(3+2i)$ as its roots is

A. $x^2-6x+13=0$ B. $x^2+6x-13=0$

C. $x^2-6x-13=0$ D. $x^2+6x+13=0$

Solutions:

Ans.: A

- **Sum of roots** $=(3-2i)+(3+2i)=6$
- **Product of roots** $=(3-2i)(3+2i)=(3)^2-(2i)^2=13$
- **A possible quadratic equation :** $x^2-6x+13=0$

Question 17:

The quadratic equation $x^2+4x=k-3$ has two distinct real roots, find the range of values of k.

A. $k<-1$. B. $k<7$.

C. $k>7$. D. $k>-1$.

Solutions:

Ans.: D

- $x^2+4x+(3-k)=0$
- $\Delta = (4)^2-4(1)(3-k) > 0$

$16-12+4k > 0$

$k > -1$

Question 18:

If $\dfrac{yi}{x+6i}=2-3i$ where x and y are real constants, then $y=$

A. -9 B. 7

C. 37 D. 39

Solutions:

Ans.: D

- $yi = (x+6i)(2-3i)$

$yi = 2x+12i-3xi-18i^2$

$0 + yi = (2x+18) + (12-3x)i$ ..... **(*)**

- **Compare real parts of both sides of (*) :**

$2x+18=0$

$x=-9$

- **Compare imaginary parts of both sides of (*) :**

$y=12-3(-9)$

$=39$

Solutions of Question 18: Ans.: D

OR

. **Rewrite the question as :** $\frac{yi}{2-3i} = x + 6i$.

. **Switch the calculator to the mode for working with complex numbers :**

A. : $\frac{-9i}{2-3i} = \frac{27}{13} - \frac{18}{13}i$

B. : $\frac{7i}{2-3i} = -\frac{21}{13} + \frac{14}{13}i$

C. : $\frac{37i}{2-3i} = -\frac{111}{13} + \frac{74}{13}i$

D. : $\frac{39i}{2-3i} = -9 + 6i$

Question 19:

If $\log(a - 3x) = 2$, then $x =$

A. $\frac{a-2}{3}$

B. $\frac{a-100}{3}$

C. $\frac{100a}{3}$

D. $10^{\frac{a-2}{3}}$

Solutions: Ans.: B

. $\log(a - 3x) = 2$

$a - 3x = 10^2$

$a - 100 = 3x$

$\frac{a-100}{3} = x$

OR

. **Put** a=**106** and x=2 satisfying $\log(a-3x)=2$!

. **Put** a=**106** into options A, B, C, D :

A. : $\frac{104}{3}$ B. : 2 C. : $\frac{10600}{3}$ D. : $10^{\frac{104}{3}}$

Question 20:

Which of the following is the greatest ?

A. 500^{2010}

B. 1020^{1500}

C. 1500^{1020}

D. 2010^{500}

Solutions: Ans.: A

(**A**) $x = 500^{2010}$; $\log(w) = 2010 \times \log(500) = 5424.930$

(**B**) $y = 1020^{1500}$; $\log(z) = 1500 \times \log(1020) = 4512.900$

(**C**) $z = 1500^{1020}$; $\log(y) = 1020 \times \log(1500) = 3239.613$

(**D**) $w = 2010^{500}$; $\log(x) = 500 \times \log(2010) = 1651.598$

Question 21:

If $4^x = a$, then $8^{x+1} =$

A. $4a^{1.5}$ B. $8a^{1.5}$

C. $4a^{-0.5}$ D. $8a^{-1.5}$

Solutions:

Ans.: B

. $4^x = a$

$(2^2)^x = a$

$(2^x)^2 = a$

$2^x = \sqrt{a}$

. $8^{x+1} = (8^x)\times(8)$

$= (2^3)^x\times(8)$

$= (2^x)^3\times(8)$

$= (\sqrt{a}\ )^3\times(8)$

$= 8a^{1.5}$

OR

. **Substitute** x=**1** giving a=**4** :

and $8^{1+1} = 64$

. **Substitute** a=**4** into options A, B, C, D :

A.: $4(4)^{1.5} = 32$

B.: $8(4)^{1.5} = 64$

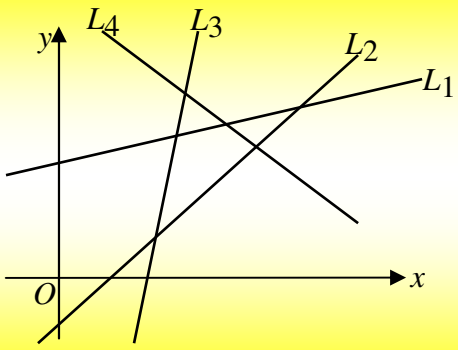
C.: $4(4)^{-0.5} = 2$

D.: $8(4)^{-1.5} = 1$

Question 22:

In the figure, L_1, L_2, L_3, L_4 are straight lines with slopes m_1, m_2, m_3, m_4 respectively.

Which of the following must be true ?



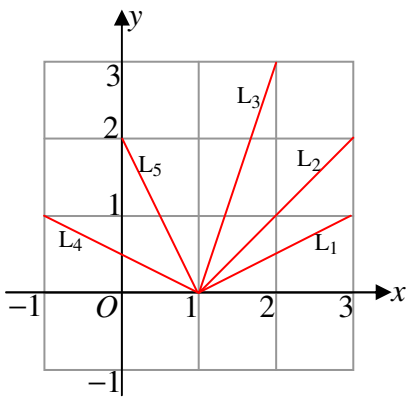
A. $m_4 > m_1 > m_2 > m_3$ B. $m_3 > m_2 > m_1 > m_4$

C. $m_2 > m_1 > m_4 > m_3$ D. $m_2 > m_1 > m_3 > m_4$

Solutions:

Ans.: B

. The relationship can be illustrated by the diagram shown :



. Slope of $L_1 = \frac{1}{2}$

Slope of $L_2 = 1$

Slope of $L_3 = 3$

Slope of $L_4 = -\frac{1}{2}$

Slope of $L_5 = -2$

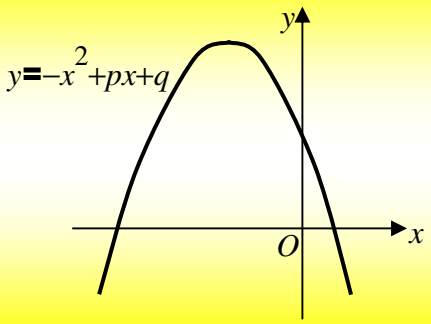
Question 23:

The figure shows the graph of $y=-x^2+px+q$.

Which of the following is true ?.

A. $p>0, q>0$ B. $p>0, q<0$

C. $p<0, q>0$ D. $p<0, q<0$



Solutions:

Ans.: C

- By considering the y-intercept of the graph,
  $q > 0$

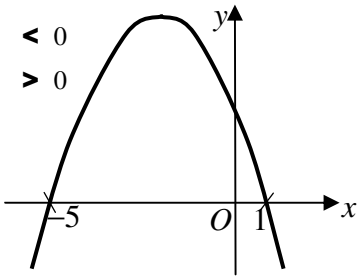
- By considering the x-coordinate of the vertex,
  $x = -\dfrac{p}{2(-1)}$
  $= \dfrac{p}{2}$
  < 0 as shown in the figure,

so $p < 0$

OR

- **Let** the equation of the graph be
  $y = -(x+5)(x-1)$
  $y = -(x^2+4x-5)$
  $y = -x^2-4x+5$

- $p = -4 < 0$
  $q = 5 > 0$



Question 24:

The figure shows the graph of $y=f(x)$.

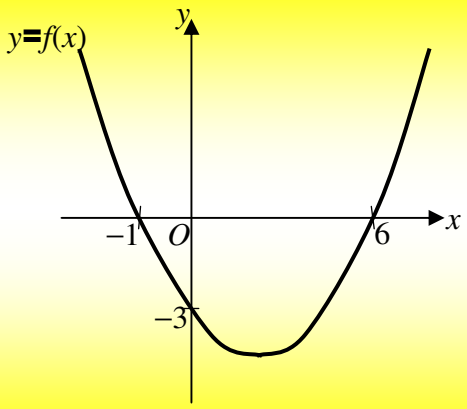
If $f(x)$ is a quadratic function, then $f(x)$ =

A. $\dfrac{1}{2}(x+1)(x-6)$.

B. $2(x+1)(x-6)$.

C. $\dfrac{1}{2}(x-1)(x+6)$.

D. $2(x-1)(x+6)$.



Solutions:

Ans.: A

- By considering the x-intercepts of the graph,
  $f(x) = a(x+1)(x-6)$

- For the y-intercept, $x=0$ and $y=-3$,
  $-3 = a(0+1)(0-6)$
  $a = \dfrac{1}{2}$

Question 25:

If $f(x)=\frac{x}{x-1}$, then $f(a)\cdot f(\frac{1}{a})$ =

A. 1

B. $\frac{1}{1-a}$

C. $-\frac{a}{(a-1)^2}$

D. $\frac{a}{(a+1)^2}$

Solutions:

Ans.: C

. $f(a)\cdot f(\frac{1}{a}) = (\frac{a}{a-1})(\frac{\frac{1}{a}}{\frac{1}{a}-1})$

$= (\frac{a}{a-1})(\frac{1}{a}) \div (\frac{1-a}{a})$

$= (\frac{a}{a-1})(\frac{1}{a})(\frac{a}{1-a}) = \frac{a}{-(a-1)(a-1)}$

OR

. **Substitute** a=**2**:

$f(\mathbf{2})\cdot f(\frac{1}{\mathbf{2}}) = ... = -2$

. **Sub.** a=**2** into options A,B,C,D:

A: 1; B: −1; C: −2; D: $\frac{2}{9}$

Question 26:

If $x>0$ and $x\neq 1$, which of the following must be true ?

(1) $\log_5 x^2 = \frac{1}{2\log_x 5}$.

(2) $\log_8 x = \frac{\log_7 x}{3\log_7 2}$.

(3) $\log_x 10 = \log x^{-1}$.

A. (1) only

B. (2) only

C. (1) and (3) only

D. (2) and (3) only

Solutions:

Ans.: B

(**1**) $\log_5(x^2) = 2\times\log_5(x) = 2\times\frac{\log(x)}{\log(5)}$

$= 2\times\frac{1}{\frac{\log(5)}{\log(x)}} = \frac{2}{\log_x(5)}$

(**2**) $\log_8(x) = \frac{\log_7(x)}{\log_7(8)} = \frac{\log_7(x)}{\log_7(2^3)} = \frac{\log_7(x)}{3\log_7(2)}$

(**3**) $\log_x(10) = \frac{\log(10)}{\log(x)} = \frac{1}{\log(x)} = [\ \log(x)\ ]^{-1}$

OR

. (**1**) **Put** x=**5** & verify by calculator.

. (**2**) **Put** x=**8** & verify by calculator.

. (**3**) **Put** x=**10** & verify by calculator.

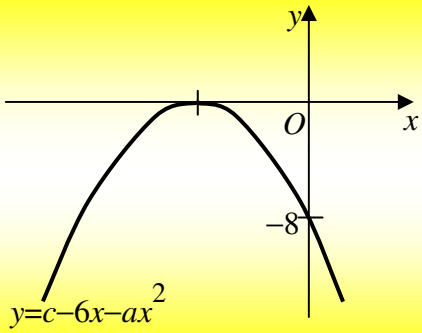
Note: $\log(x^{-1}) \neq [\ \log(x)\ ]^{-1}$

Question 27:

In the figure, the graph of $y=c-6x-ax^2$ touches the x-axis. Find a .

A. $-\frac{8}{3}$ B. $-\frac{9}{8}$

C. $\frac{9}{8}$ D. $\frac{8}{3}$



Solutions:

Ans.: C

- By considering the y-intercept of the graph of $y=-ax^2-6x+c$,
  $c = -8$
- Equation of the graph becomes $y=-ax^2-6x-8$,
- For the graph of $y=-ax^2-6x-8$ touches the x-axis,
  we need

$$\Delta = (-6)^2 - 4(-a)(-8) = 0$$
$$36 - 32a = 0$$
$$a = \frac{9}{8}$$

OR

- A. **Put** $a=-\frac{8}{3}$ & verify by calculator.
- B. **Put** $a=-\frac{9}{8}$ & verify by calculator.
- C. **Put** $a=\frac{9}{8}$ & verify by calculator.
- D. **Put** $a=\frac{8}{3}$ & verify by calculator.

Question 28:

Which of the following statements about the graph of $y=(x+2)^2-9$ is true ?

A. The coordinates of the vertex of the graph are $(-2, 9)$.

B. The equation of the axis of symmetry of the graph is $x=2$.

C. The x-intercepts of the graph are -1 and 5 .

D. The y-intercept of the graph is -5 .

Solutions:

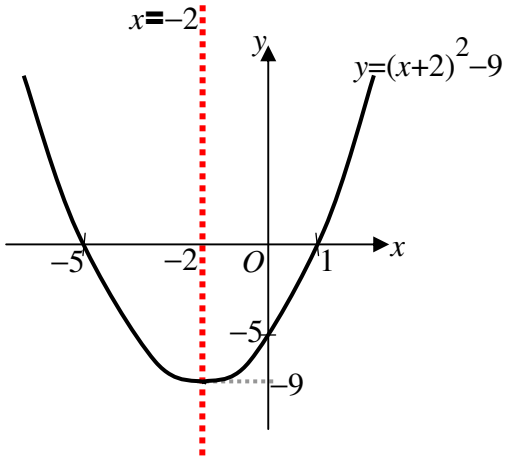
Ans.: D

A: The vertex of the graph = $(-2, -9)$

B: The equation of the axis of symmetry is $x=-2$.

C: For $y=0$, $(x+2)^2=9$, $(x+2)=3$ or -3, $x=1$ or -5 .

D: For $x=0$, $y=(0+2)^2-9=-5$.



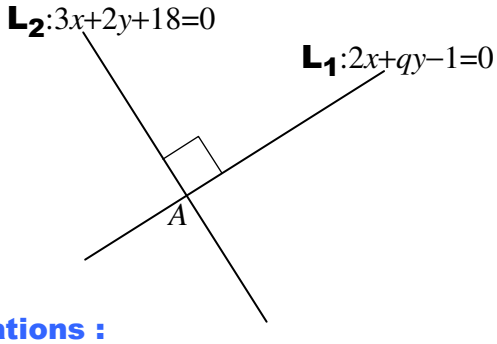
Question 29:

If the straight lines $2x + qy - 1 = 0$ and $3x + 2y + 18 = 0$ are perpendicular to each other and intersect at the point A, find the coordinates of A.

A. (4, 3) B. (–4, 3) C. (4,–3) D. (–4,–3)

Solutions:

Ans.: D



- For L_1 : slope $= m_1 = -\frac{2}{q}$
- For L_2 : slope $= m_2 = -\frac{3}{2}$
- For $L_1 \perp L_2$, $(-\frac{2}{q})(-\frac{3}{2}) = -1$,

  $q = -3$
- For A : **solving** the **simultaneous equations :**

  $2x - 3y - 1 = 0$ … **(1)**

  and $3x + 2y + 18 = 0$ … **(2)**

  gives $x = -4$

  $y = -3$; $A = (-4,-3)$

Question 30:

The figure shows the graph of the straight line $px - qy + 12 = 0$. Which of the following are true ?

(1) $p > 0$.

(2) $p < 4$.

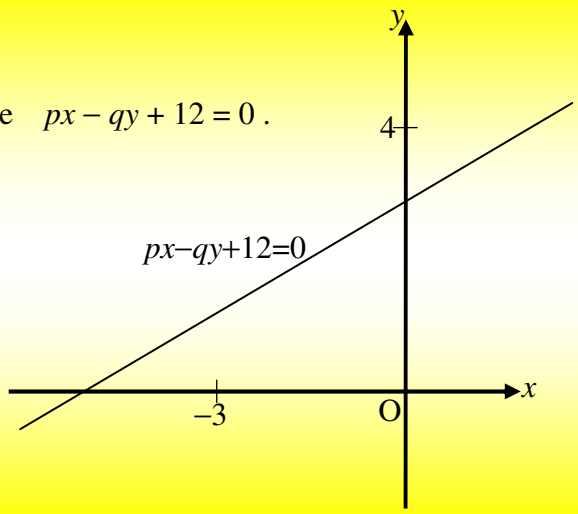
(3) $q < 3$.

A. (1) and (2) only

B. (1) and (3) only

C. (2) and (3) only

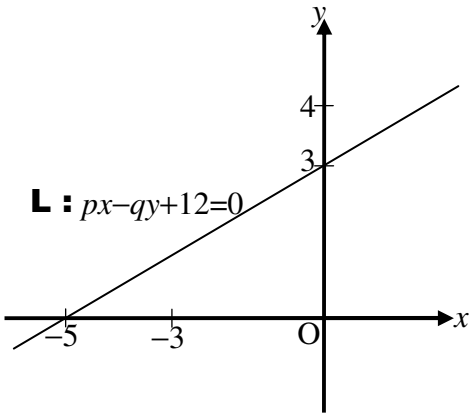
D. (1), (2) and (3)



Solutions:

Ans.: A

- Let the x-intercept and y-intercept of the straight line **L:** $px-qy+12=0$ be (–5,0) and (0,3) respectively as shown.



- Equation of **L** can be obtained as :

  $\frac{y-3}{x-0} = \frac{0-3}{-5-0}$

  $3x - 5y + 15 = 0$

  which can be rewritten as

  $2.4x - 4y + 12 = 0$

- p=2.4 , q=4